Line-Wise Conics in Projective Geometry

# Conic Transformation Drawings

(Instructions for the drawings that follow)

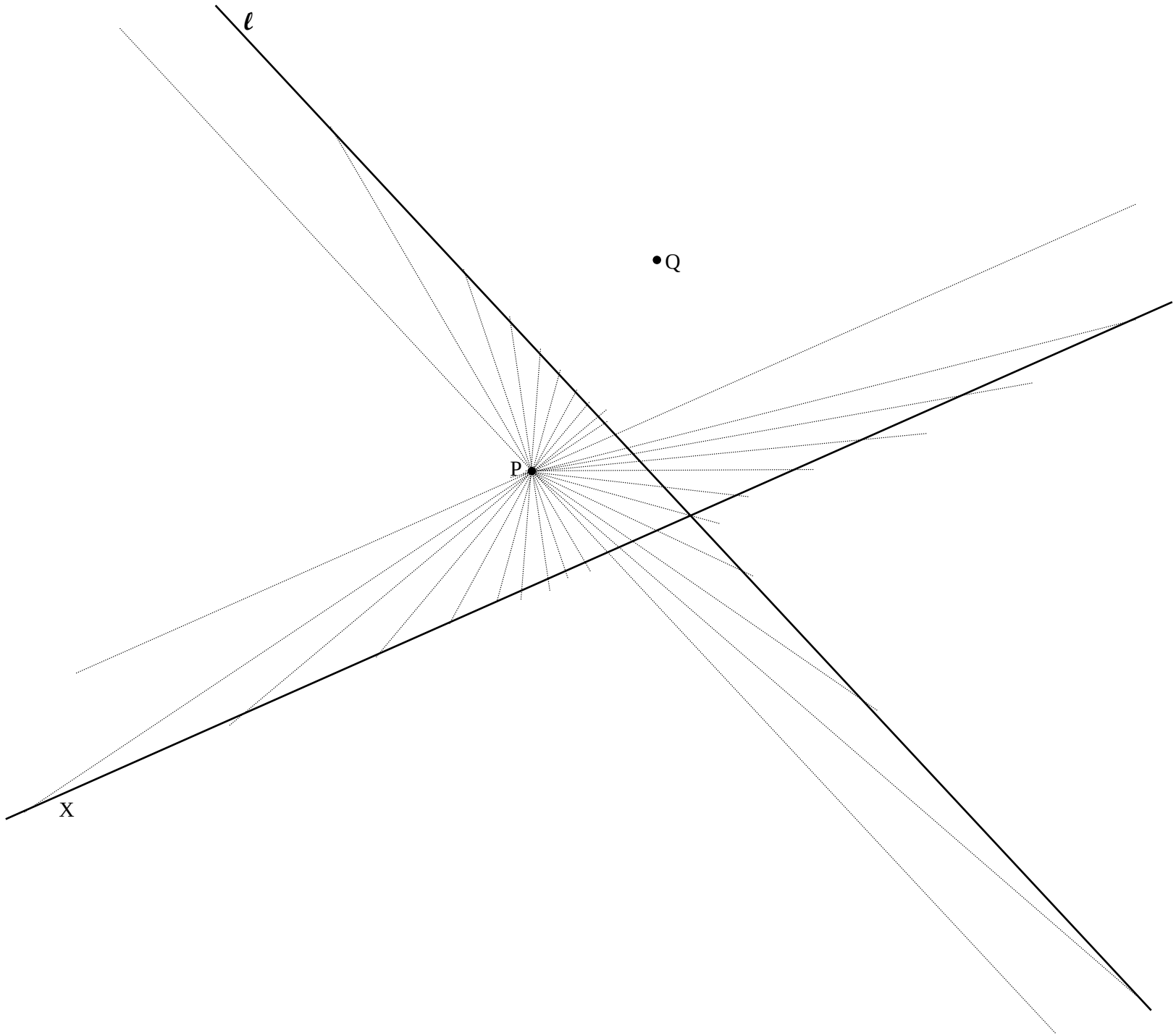
## Instructions (For the teacher)

- The following progression of drawings put into practice Rudolf Steiner's suggestion that whenever possible it is good to have the students picture geometric form in movement. In the case of the drawings shown here we are able to see an ellipse become transformed into a parabola and then into a hyperbola.
- It is best, of course, to have the students experiment first with the line-wise conic drawings, and making them from "scratch".
- Although there are different methods of constructing a line-wise conic, the method I tend to use is as follows:
  - We are given three fixed lines X, Y and $\ell$, and two fixed points P and Q. ) Notice that in the case of the following drawings, the only difference between each drawing is the position of line Y. In fact, the whole point of this exercise is to visualize that line Y is in movement, and this in turn causes the conic section to also be in movement.)
  - First we do a perspectivity of the several points on X to points on l, using P as the point of perspectivity. (This part has already been done with each of the drawings in order to get the students started. However, it is helpful to label these points as 1..20 on each of the two lines so that they correspond according to the perspectivity.)
  - Then we do a perspectivity of these newly marked points on l to points on Y, using Q as the point of perspectivity.
  - Now we draw the "connector lines" by connecting corresponding points on lines X and Y (e.g., connect point 7 on line X to point 7 on line Y). I find it best to draw these connector lines in color. These connector lines should very nicely form the boundary of a conic section.
  - The following four drawings only show four "freeze-frames" of line Y as it is moving down the page. Of course, we could very well construct the resulting curve (i.e., conic section) for many other locations of line Y. I could even imagine a classroom of 20 students each constructing a conic with Y in a different position –although I would tend to keep it all consistent with the four drawings shown here and therefore have line Y horizontal on the page.

Projective Geometry

# Conic Transformation

(Template – line y needs to be drawn)



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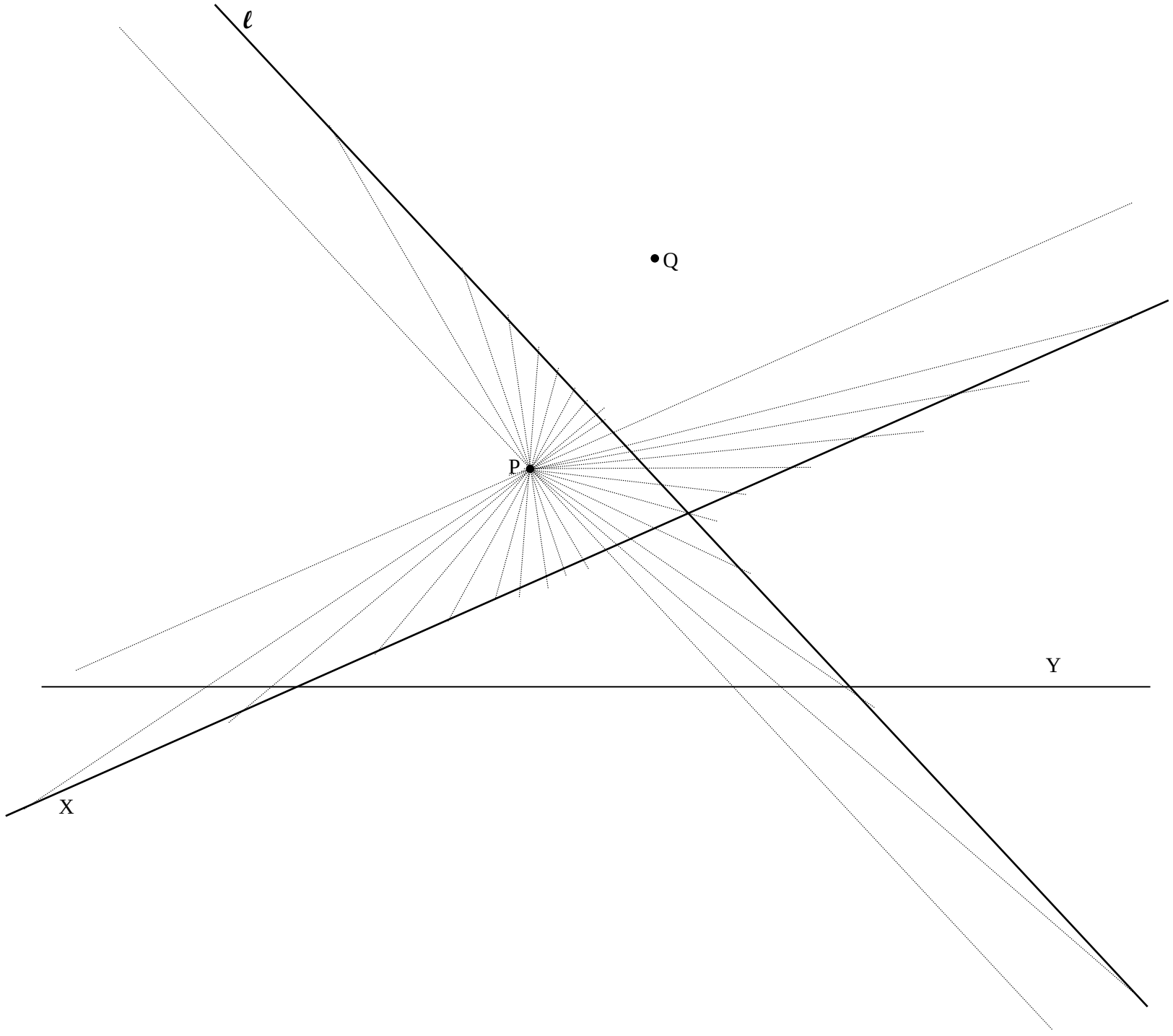
# Conic Transformation – Drawing #1



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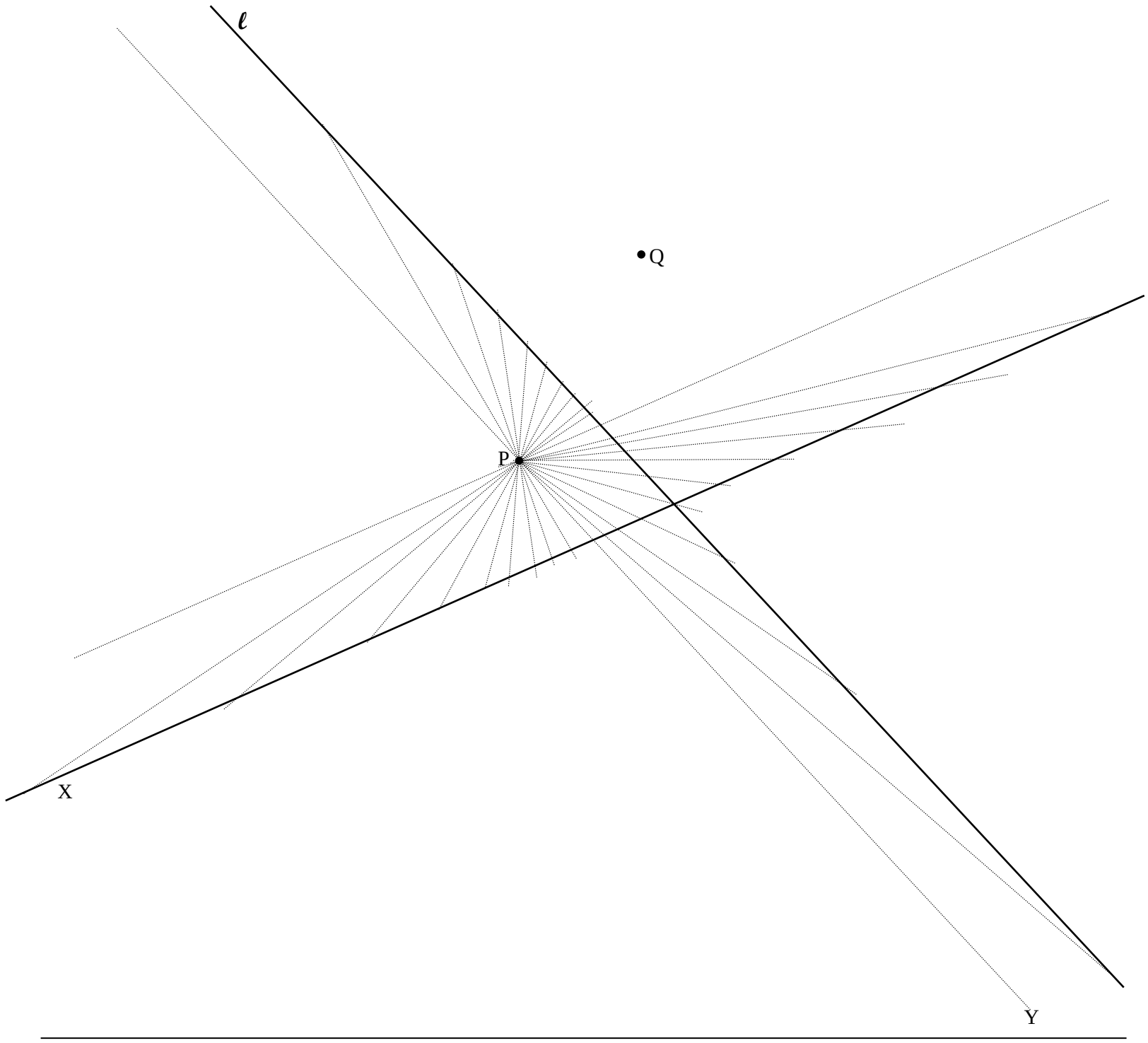
# Conic Transformation – Drawing #2



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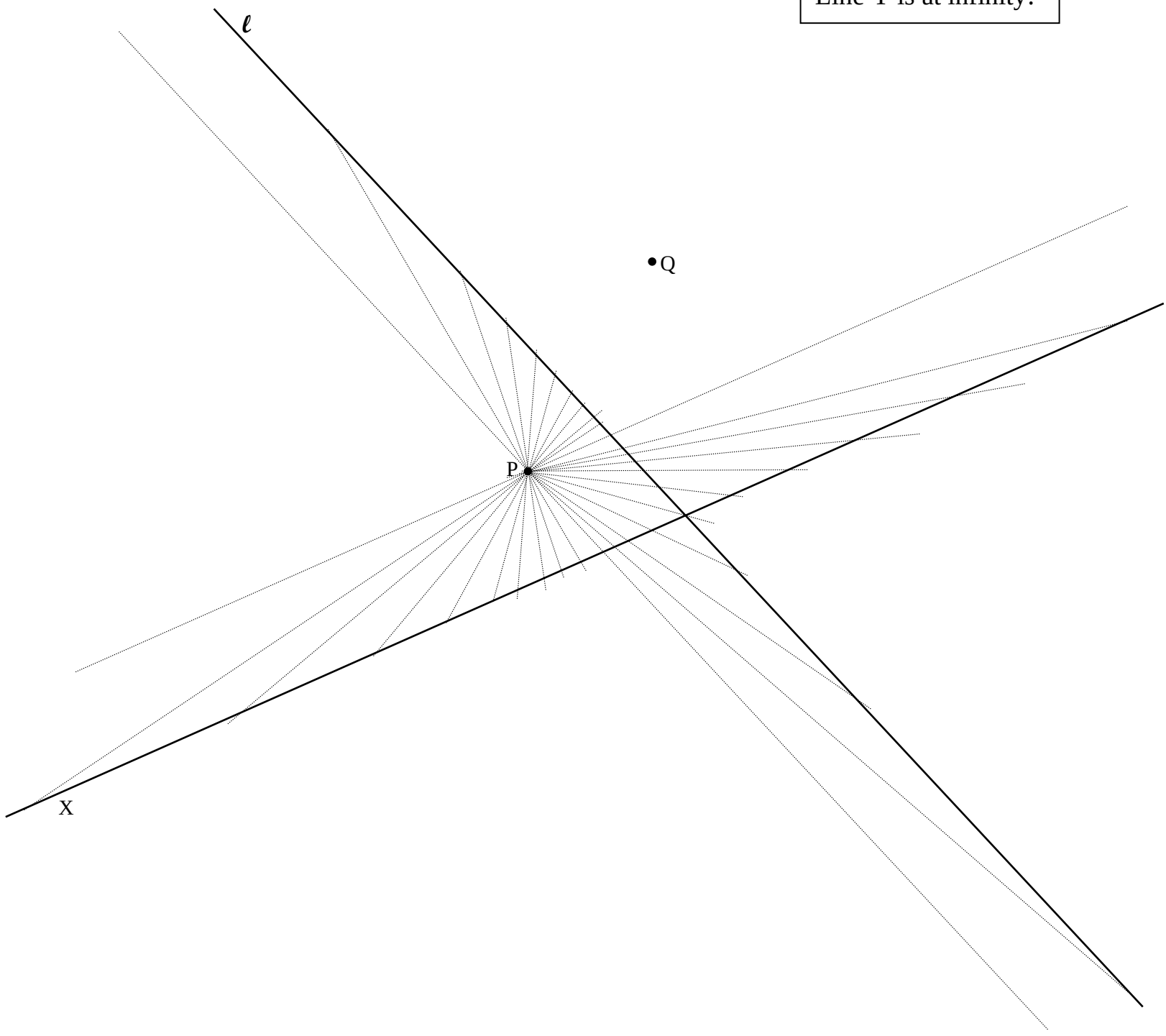
# Conic Transformation – Drawing #3

Line Y is at infinity!



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# Conic Transformation – Drawing #4