The Grains of Rice Problem

The Story:

There was a king in India who loved mental games and puzzles. A wise, but poor man in his kingdom invented the game of chess for him. The king enjoyed the game so much that he invited the wise man to his castle and told him that as a reward he could have anything in the kingdom that he desired. The wise man thought for a moment and then said that since his village sometimes did not have enough food, that he would like a good amount of rice.

When the king asked him how much rice he would like, the wise man stated his answer as a puzzle. He said that a single grain of rice should be placed on the first square of the chessboard (which has a total of 64 squares). Then two grains of rice should be placed on the second square, and then double that amount (4 grains) on the third square, and double that amount (8 grains) on the fourth square, and so on up to the last square. That is how much rice he would like – if the king didn't feel that the request was too great. He warned the king that all the rice wouldn't fit nicely on the chessboard, but that didn't really matter – he just wanted that amount of rice. The king thought to himself that the wise man was actually quite a fool since he could have had anything in the kingdom, and he was only requesting a few bags of rice.

How much rice was the wise man requesting?

(The second part of the story should be told only after the students have worked the problem out for themselves.) The king ordered his servants to go into the royal kitchen and carry out the wishes of the wise man. However, once they reached the 23rd square, they came to the king and told him that they had run out of rice in the kitchen. The king was surprised, but told them to take as much rice as needed from the royal granary. The servants worked for two whole days bringing rice from the granary. But when they reached the 39th square of the chessboard they approached the king and told him that they had emptied the entire royal granary and had only reached the 39th square. The king was quite shocked. He asked how much more rice would be needed, and the royal astronomer said that he had calculated that it was far more rice than had ever been produced in the entire world. The king laughed when he realized that he had been tricked. He then approached the wise man and asked him what he would really like to have, and the wise man said that really his greatest wish was to marry the king's daughter. The king thought that the wise man was worthy, and so the marriage took place on the next day.

The Questions:

1. How many grains of rice are there on the whole chessboard (assuming that it would somehow fit)?
2. How many 25-pound sacks of rice would this be, and if all the sacks were laid in a line end-to-end, how far would they stretch? (Assume that each sack is 20 inches long and contains around 400,000 grains of rice.)
3. What is the volume of the rice? (Assume that there are 400 grains of rice in a tablespoon.)

Solutions:

1. The number of grains on the last square is 2^{63}. The total number of grains on the whole board is $2^{64}-1$ or 18,446,744,073,709,551,615 (**see Appendix E,** *Powers of Two Table*).
2. Rounding this to 18,400,000,000,000,000,000 and dividing by 400,000 gives us approximately 46,000,000,000,000 (46 trillion) sacks of rice. Since each sack is 20 inches long, the length of the line of sacks works out to be about 920,000,000,000,000 inches, which is 76,666,666,666,666 feet. We can round this figure and then divide by 5280 (the number of feet in a mile) to get 14,500,000,000 miles, which is about 156 times longer than the distance to the sun!

3. The volume is calculated as follows: There are 7.48 gallons in a cubic foot, so we calculate the number of grains in a cubic foot as: $\frac{400 \text{ grains}}{\text{tbsp}} \cdot \frac{2 \text{ tbsp}}{\text{fl.oz.}} \cdot \frac{128 \text{ fl.oz.}}{\text{gal}} \cdot \frac{7.481 \text{ gal}}{\text{ft}^3} \approx 766,000 \frac{\text{grains}}{\text{ft}^3}$

   The total amount of rice is then approximately 24,000,000,000,000 cubic feet, and since there are about 147,000,000,000 cubic feet in a cubic mile, the volume of all the rice is approximately 163 cubic miles! This is also approximately 890,000 boxes that are 100yd x 100yd x 100yd, each box having the approximate volume of a large football stadium.