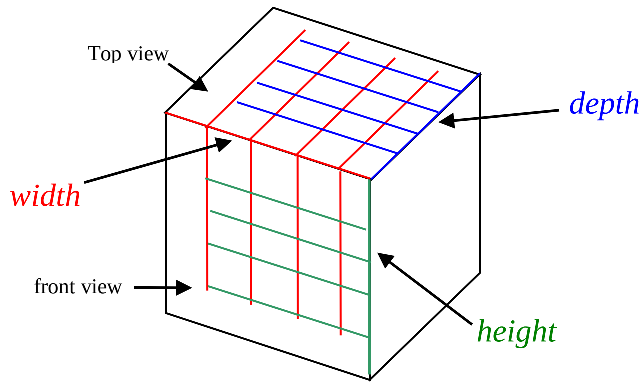
The Complete Solution of Descriptive Geometry:
Finding any arbitrary auxiliary view

The final question is: How can we describe *any arbitrary* view of the object?

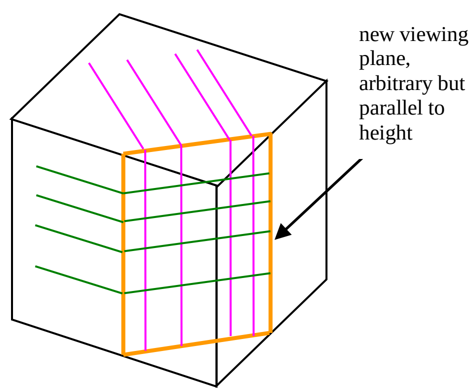
Summary of our work so far:

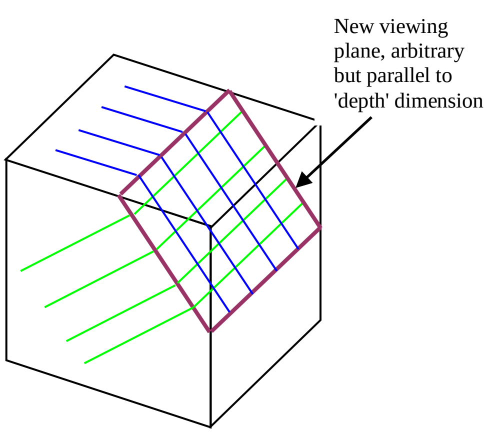
1. **Any two orthogonal adjacent views completely describe the locations of the object points in space.** Think first of the top view and front view, for example: the top view completely describes the width and depth positions, while the front view describes the height and width. These two adjacent orthogonal views, which share one dimension, completely locate the points in all three dimensions of space.



2. The side view is easy to construct now. The image of the side viewing plane is a line in both the top view and the front view, since that plane is orthogonal to both. By projecting the spacing along the dimension out to the right onto the line representing the side view we locate the widths for that view. By projecting the spacing along the heights of the front view onto the line representing the side view plane, we locate the heights for the side view.

3. **A view at an arbitrary rotation about a vertical axis, is just like the side view.** We noted that any view between the front and side, or any view that is a similar vertical sectioning of the box (parallel to the *height* dimension), will also appear as a line in the top view. We can thus choose an arbitrary vertical plane and project the points onto the line representing it in the top view. This locates the widths for the new view. The heights are the same as for the front or side (all such vertical viewing planes have the same heights).

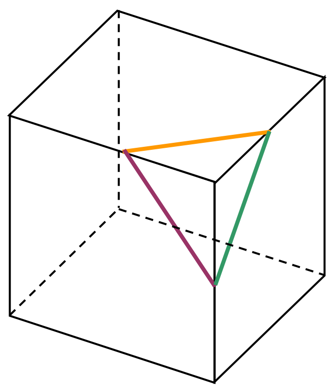


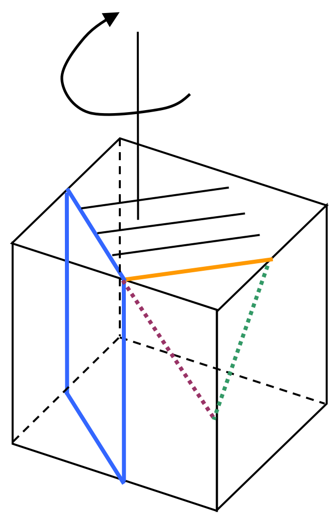


4. **The same idea extends to an arbitrary rotation about a horizontal axis.** This same procedure works for any arbitrary horizontal sectioning, that is, a plane that is parallel to the *depth* dimension for example. These appear as lines in the front view, and projecting the points onto that line locates the new heights for the desired view. The widths will all be the same as those in the side view. In fact all sections parallel to the depth dimension have the same widths (those that are the same as the side view).

The complete solution

Finally, the question is: ***how can we construct a view that is at arbitrary angles to all the initial dimensions?*** The solution is a simple matter of reducing the problem to one of the previous two situations (#3 or 4). We can achieve this if we can get our section to be perpendicular to a view that we can make. For example, we could choose a "new" front view such that it is perpendicular to the section slice as seen in the top view. Then the section will appear as a line in that new front view onto which we can project the new heights for the desired final view. It is as if we rotate our box so that we force the box to line up with the section slice to reduce it to the previous condition (#4)!

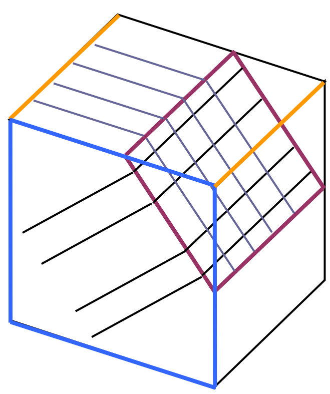




Imagine rotating our projection box to align the "depth" with the section slice. Orthogonal to that is a "new front" view

Seen in our new front view, the slice is orthogonal and appears simply as a line.

New 'front' view



By choosing a front view at 90º to one direction of the section slice, we have reduced the problem to one we know how to solve easily.

In this way, the method of Gaspard Monge achieves the solution to any view of an object in at most two steps given the initial two orthogonal adjacent views.